

ARE THERE TOPOLOGICAL BLACK HOLE SOLITONS IN STRING THEORY?

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February 1987

published in Gen. Rel. Grav. 19 (1987) 1173.

Abstract

We point out that the celebrated Hawking effect of quantum instability of black holes seems to be related to a nonperturbative effect in string theory. Studying quantum dynamics of strings in the gravitational background of black holes we find classical instability due to emission of massless string excitations. The topology of a black hole seems to play a fundamental role in developing the string theory classical instability due to the effect of sigma model instantons. We argue that string theory allows for a qualitative description of black holes with very small masses and it predicts topological solitons with quantized spectrum of masses. These solitons would not decay into string massless excitations but could be pair created and may annihilate also. Semiclassical mass quantization of topological solitons in string theory is based on the argument showing existence of nontrivial zeros of beta function of the renormalization group.

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‡ Gravity Research Foundation Essay for 1987.

It is believed that black holes should play a very important role in quantum gravity. The most remarkable theoretical prediction about black holes, which also opened the beautiful connection with thermodynamics is the Hawking effect.^{1,3} Hawking discovered that a black hole will radiate quantum mechanically with a thermal spectrum having a temperature related to the hole's mass M , angular momentum L , and the electric and magnetic charges Q and P .

Classical general relativity and the physically reasonable assumption of non-negativity of energy imply that the event horizon of a black hole has the topology of a sphere S^2 . It is plausible to assume that black holes “survive” quantization of gravity and they should be present also in the future theory of all interactions. At least it is so in the (super) string theory.⁴ A black hole is the highly nontrivial and nonperturbative solution of the Einstein equations which is classically stable and unique.² It is impossible to obtain a black hole solution by summing up a finite number of terms in the expansion of Einstein's equations around Minkowski vacuum, a fact which is demonstrated by the fact that the topology of spacetime outside the event horizon, or, what is the same, the topology of the Euclidean black hole is $R^2 \times S^2$. In the following we will define a black hole in semiclassical or quantum gravity by saying that the second homology group of its manifold is nontrivial, $H_2(M_{bh}) = Z$.

What are the implications of this beautiful mathematical fact in quantum gravity? It seems that the physical implications of this simple fact are not quite well understood. The condition of the nontrivial second homology group $H_2(M_{bh})$ for black holes is reminiscent of the nonabelian t'Hooft-Polyakov magnetic monopoles which are present if the second homotopy group of a certain manifold is nontrivial.

In particular it seems unclear if there is some relation between the fundamental quantum mechanical instability of black holes and the fact that $H_2(M_{bh}) = \mathbb{Z}$. One may ask, is it possible to stabilize a black hole with respect to the Hawking effect by introducing topological charges which would bound the energy of a hole from below (the Bogomolny'i bound)? If it were possible, then there would exist topological black hole solitons. The usual way to understand the Hawking effect in QFT does not rely on the fact that the topology of an event horizon is S^2 . Rather, it is important that there be a horizon and that the Bekenstein-Hawking entropy be $S_{bh} = \frac{A}{4}$, where A is the area of the event horizon. It seems that this property of black hole physics, which attributes an intrinsic entropy to a black hole and equals it to a basic geometrical quantity—the area A of the generator of the second homology group of a black hole manifold—should be derived from fundamental principles of quantum gravity. Accepting the information theory point of view on entropy, advocated particularly by Bekenstein⁶, we reach the conclusion that the number of ways a black hole can be “built up” depends on its geometrical properties only: $\Gamma = e^{S_{bh}} = e^{A/4}$.

I think that there should exist the way to understand the relation between the topological properties of black holes, like $H_2(M_{bh}) = \mathbb{Z}$, their entropy and fundamental quantum mechanical instability. The Hawking effect should manifest its presence in the future unified theory of gravity and all fundamental interactions.

One of the promising candidates for such a theory is theory of strings.⁴ What is attractive in this theory is the presence of a massless spin-2 particle in its perturbative spectrum. The tree level interactions of this “graviton” are effectively described by the Einstein-Hilbert action. In the classical limit of large occupation

number of gravitons we have a nontrivial macroscopic graviton “condensate” $\Phi_{\mu\nu}$, and we can define the effective metric $g_{\mu\nu} = \eta_{\mu\nu} + \Phi_{\mu\nu}$ which is a solution of the Einstein equations. The conformal invariance implied by the geometrical fact that the string world-sheet is two-dimensional is highly restrictive in QFT.⁴ In the low energy limit only the massless string excitations play a role and we can study the dynamics of the quantum string propagating in the classical background of the massless field “Bose condensates”. It is definitely interesting to address the question of how the quantum dynamics of strings is restrictive for black holes. The closed string contains other massless excitations: a spin-0 dilaton and a spin-0 abelian two-form $B_{\mu\nu}$, which plays a fundamental role in anomaly cancellations. One may expect that the Hawking effect will manifest itself in quantum mechanical inconsistency of string propagation in the gravitational field of a black hole. We would like to demonstrate in this essay how the nontrivial topology of black holes implies their instability in string theory. The same sort of argument leads to the conclusion that there are possible topological black hole solitons stabilized by the topological “quantum numbers” and they would be solutions of string equations of motion. They would be created or annihilated in pairs. Also, what seems to be the fundamentally new aspect of black hole physics brought about by string theory, the hole’s mass or irreducible mass $\left(\frac{A}{16\pi}\right)^{1/2}$ is quantized. This sort of behavior of Planck’s mass black holes was postulated several years ago by Bekenstein.^{3,6} The famous Bekenstein-Hawking entropy formula obtains for such objects with the simplicity of combinatorics.

Let us now go to the details and see how string theory makes use of the nontrivial black hole topology. The string interaction with background fields is described

by the nonlinear sigma model

$$I = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h} (h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X) + \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X) + \frac{1}{2} R\Phi(X)), \quad (1)$$

where $h_{\alpha\beta}$ is the string world sheet metric and $(2\pi\alpha')^{-1} = M_S^2$ is the string tension. Considering only the gravitational background and demanding that the two-dimensional nonlinear sigma model (1) be quantum mechanically conformally invariant leads, in the lowest approximation, to the classical Einstein equations for the background metric.

Classical string equations of motion are derived from the condition of conformal invariance of quantum dynamics of the sigma model—the sigma model should be Weyl invariant on the quantum level! Black holes are solutions of the string equations of motion in the perturbative expansion in α' . Is this also true non-perturbatively? The answer is no, because black holes are unstable due to the Hawking effect; we conclude that instability of black holes in string theory is a genuinely nonperturbative effect in α' ! Let us see how it occurs. String theory gives us the unique opportunity of including topological issues of quantum gravity into the game. Breakdown of Weyl invariance or beta function depends only on the short-distance behavior of the QFT and does not depend on the string world-sheet topology. The Euclidean path integral $Z = \int DX Dh \exp(-I)$ can be calculated on S^2 , when the Weyl invariance holds, which corresponds to the classical approximation to string theory. The condition for a classical solution in string theory is $\langle V \rangle = 0$, where V is the vertex operator of a given physical state. It means that the vacuum is stable with respect to emission of particles corresponding to V . In order to study stability of a given ground state it is sufficient to check it at

the tree level, which corresponds to the classical approximation of string theory. The last condition corresponds at the tree level to the world-sheet conformal invariance. Conformal invariance on S^2 means, in particular, invariance under the scaling $z \mapsto \lambda z$ on the complex plane. A physical closed-string vertex operator has dimension two and transforms as $V \mapsto \lambda^{-2}V$, which implies $\langle V \rangle = 0$. V should be evaluated only at zero momentum, which is on shell only for massless particles. If $\langle V \rangle \neq 0$ for massless particles, it means that a given classical ground state is unstable with respect to emission of “soft” particles into the vacuum. If it happens for black holes it means that the Hawking effect is a semiclassical phenomenon in string theory.

Consider now a black hole background on which a quantum string propagates which has a characteristic length scale $R = \left(\frac{A}{4\pi}\right)^{1/2}$; A is the area of the event horizon. It is convenient to rescale the sigma model fields $X^\mu \mapsto RX^\mu$ introducing dimensionless fields and the action

$$I = \frac{1}{2g^2} \int d^2\sigma \partial_\alpha X^\mu \partial^\alpha X^\nu g_{\mu\nu}(X), \quad (2)$$

where $g^2 = \frac{\pi\alpha'}{2R^2}$ is the dimensionless sigma model coupling constant. The condition for conformal invariance of (2) is the vanishing of the beta function: $\beta(g^2) = -(\mu \frac{\partial}{\partial \mu})g^2(\mu)$. One may ask if the Schwarzschild black hole is a solution of the string equations of motion or, equivalently, if it is stable. Of course we assume that the $d = 10$ dimensional string model is compactified on some compact manifold K such that its beta function is vanishing, which seems to restrict the possible compactifications $M_{bh} \times K$. The sigma model (2) on a black hole background has nontrivial instantons due to the fact that $H_2(M_{bh}) = \pi_2(M_{bh}) = \mathbb{Z}$ is nontrivial

and there exist maps $X^\mu(\sigma)$ such that I is finite and bounded from below. A historic role of instantons is to violate classically valid symmetry or produce effects which couldn't be seen at any finite order of perturbative expansion. So one may ask, what is the effect of world sheet instantons on a black hole background?

The effect we find is destabilization of a black hole. We can evaluate the partition function for the sigma model on the Schwarzschild black hole background in the instanton (WKB) approximation. In this approximation we can also tell what the beta function is. The Euclidean rescaled Schwarzschild metric in Kruskal coordinates is

$$ds^2 = g_{\mu\nu}dX^\mu dX^\nu = f dud\bar{u} + r^2(1 + w\bar{w})^{-2}dw d\bar{w}, \quad (3)$$

$w = \cot(\theta/2)e^{i\phi}$, $u = (r-1)^{1/2}e^{(r+2ikt)/2}$, $f = r^{-1}e^{-r}$, and k is the surface gravity $k = \frac{1}{4M}$.

Instantons are complex (holomorphic or antiholomorphic) maps from S^2 to the u or w -plane. Only $u = \text{const}$ instantons contribute to the partition function Z , because they only have finite action. Small oscillations around the w -sector $CP(1)$ instantons lead to the classical Coulomb gas partition function, a standard result of the $CP(1)$ sigma model. Small oscillations around $u = \text{const}$ give rise to a contribution depending on $r = \text{const}$ and the integral in the collective variable r has the effect that only $r = 1$ contributes significantly to the path integral ($r = 1$ is a saddle point!). Basically, we would expect that the main contribution to the string partition function would come from strings “living” close to the event horizon. A similar phenomenon was discovered by Thorne and Zurek in their statistical derivation of $S_{bh} = \frac{A}{4}$. Now we employ the fact that the dynamics of strings

in a black hole background is governed by the $CP(1)$ sigma model on the event horizon (here is where the topology of a black hole enters) and observe that it does have a nonvanishing beta function. The $CP(1)$ sigma model is asymptotically free, which means that $g = 0$ is an ultraviolet fixed point corresponding to $R \mapsto \infty$ or $M_{bh} \mapsto \infty$. It means that only ultraheavy black holes are stable in classical string theory. For very large black hole masses the Hawking effect can be neglected or, equivalently, a black hole metric is a solution of the classical string equations of motion. This dual point of view reflects the fact that the Hawking effect seems to correspond in string theory to the instability of a black hole due to massless particle emission.

Is it possible to produce an example of vanishing beta function for nonzero g ? In QFT it would correspond to the absence of the Hawking effect. It is only possible when the Bekenstein-Hawking temperature vanishes. Quantum mechanically stable black holes do exist and they are the extremal charged Kerr black holes. For simplicity we consider only the Reissner-Nordström case: $M_{bh} = M_{Pl}(Q^2 + P^2)^{1/2}$. The physical interpretation of the vanishing temperature is quite simple because we have a state, stabilized by the absolute charge conservation, behaving like a soliton. However, there is a problem with interpreting the quantum mechanically stable black holes as solitons. The mass of a soliton is semiclassically quantized. We would like to argue that string theory offers a natural mechanism for semiclassical mass quantization of black holes.

Let me give here first an ad hoc but instructive demonstration of how the quantization of mass of the extremal Reissner-Nordström dyons can occur. Assume that the theory has magnetic monopoles, which are in fact dyons (this happens in

string theory due to the topological effects of the B field⁴) satisfying the Dirac-like quantization condition: $QP = n/c$, c an integer.⁴ Then a black hole mass is: $M_{bh} = M_{Pl}(Q^2 + \frac{n^2}{Q^2 c^2})^{1/2}$. However, the ground state of a soliton should have a minimal energy; minimization of M_{bh} gives: $Q^2 = n/c$, $Q = P$, $M_{bh}^2 = 2M_{Pl}^2 n/c$. We observe that *the classical formula for a black hole dyon mass is duality invariant* but quantum effects need not respect this classical symmetry. The irreducible Christodoulou mass of a semiclassically quantized R-N dyon and its Bekenstein-Hawking entropy are: $M_{ir} = (A/16\pi)^{1/2} = M_{Pl} (\frac{n}{2c})^{1/2}$, $S_{bh} = 2\pi n/c$. The number of internal states which give rise to this entropy is:⁶ $\Gamma = e^{S_{bh}} = e^{\frac{2\pi n}{c}}$. Now we may ask in how many possible ways we can produce the mass level with the topological “quantum number” n ? A simple combinatorics gives $\Gamma = 2^{n-1}$, from which the information theory entropy is $S = \ln \Gamma = (n-1)\ln 2$. Discarding an additive constant these two entropies naively agree if $\frac{2\pi}{c} = \ln 2$, or $c = \frac{2\pi}{\ln 2} = 9.03$. For this value of c the lowest lying soliton state would have a mass $M_{bh} = .5M_{Pl}$. This quite crude argument suggests that there may exist topological black hole solitons with masses of the order of M_{Pl} which are stable quantum mechanically with respect to emission of massless particles not carrying the same charges as solitons. Such topological solitons may be created and annihilated in pairs with opposite charges, but these processes can be only studied in full quantum gravity. It is amusing to observe that the Dirac-like quantization condition for fractional charges arises in string theory where c is simply, as Witten observed, the order of the fundamental group of K . The topological properties of a compact manifold K have implications for low energy physics. In particular the Euler characteristic in some models is related to the number of generations of fundamental fermions.⁴

The nontrivial first homotopy group of K reduces the Euler characteristic c times, where c is the rank of $\pi_1(K)$. Is it only a coincidence that the mass spectrum of black hole solitons is related to the topological properties of a compact manifold K which occurs in string theory compactifications?

It is important to recognize that string theory offers the possibility of stable topological black hole solitons, which by definition are objects satisfying string equations of motion—the sigma model beta function vanishes for these configurations!

How do we get a vanishing beta function in QFT? It is a very rare situation when the beta function can be calculated exactly and has isolated zeroes. The topological effects of the B gauge field may lead to an effectively free field theory in two dimensions. Such an effect is known and a beautiful example was discovered by Witten⁵—the nonlinear sigma model on a group manifold with the Wess-Zumino term. For certain values of the coupling constant $g^2 = \frac{4\pi}{n}$ the sigma model is equivalent to a theory of free fermions and the beta function has zeroes at these values of g ; n is the quantized coupling constant of the topological WZ term. In fact, there is a nontrivial Wess-Zumino term on the black hole background. Its existence is strictly related to the fact that the horizon of Euclidean black holes has $S^2 \times S^1 = Y$ topology. There is a WZ term if the third homology group of Y is nontrivial: $H_3(Y) = \mathbb{Z}$. It means also that there is a closed but not exact form on Y which locally can be written as dB . An example is easily constructed: $dB = dt \wedge \omega$, where t is an “angle” on S^1 and ω is a volume 2-form on S^2 . B is then given locally as $t\omega$. This leads to quantization of the coefficient of the WZ term. By standard arguments the topological coupling is unrenormalized. It means that

when the “metric” coupling g is renormalized it depends on the “bare” coupling g_0 and the integer WZ coupling constant n . It may happen then that the beta function will have isolated zeros which asymptotically are $g^2 \sim 1/n$. This would lead to the quantization condition for the characteristic length scale of a black hole: $R^2 \sim n$. But now we have two mass scales in the problem: M_{Pl} and M_S .

Concluding, we would like to point out that the Hawking effect seems to be related to a semiclassical effect in string theory but is completely nonperturbative in α' due to instantons. The $R^2 \times S^2$ topology of black holes plays a fundamental role in establishing their instability in string theory. String theory allows for a qualitative description of black holes with small masses and seems to predict topological black hole solitons which would not decay into string theory massless “mesons”. One may expect that those objects have a size comparable to the compactification scale and therefore would be effectively 10-dimensional.

This research was supported by NSF grants PHY 83-18350 and PHY 86-12424 with Relativity Group at Syracuse University.

Note added on December 18th, 1996.

It makes no sense for me to list here the numerous papers which have appeared in the last decade on the subject of strings and black holes, “axionic black holes”, “quantum hair”, “world-sheet instantons”, exact solutions in the multidimensional context, etc.. Instead, I would like to suggest that the basic assumptions about the subject of so-called black hole entropy in the context of strings be thought over again. Does Nature really require strings and in what limit?

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